



1. For his summer camp, Mike packed two pairs of shoes, two pairs of shorts, and six T-shirts. In how many ways can he choose an outfit for a day at the camp? (Mike's outfit consists of a pair of shorts, a T-shirt, and a pair of identical shoes).

2. For her summer camp, Alice packed four pairs of shoes, two skirts, five blouses, and three dresses. In how many ways can she choose an outfit for a day at the camp? (Alice's outfit consists either of a skirt, a blouse and a pair of identical shoes or of a dress and a pair of identical shoes.)

3. Bella and her mom are buying a birthday gift for Max at a toy store. The store has 3 types of knight figurines, 3 types of horses, 2 types of pirate figurines, and 1 type of toy boat. Bella is allowed to choose either a knight and a horse, or a pirate and a boat. In how many ways can Bella choose Max's gift?

4. Marcus has a bag of green marbles, a bag of red marbles, and a bag of blue marbles. In how many different patterns can he place two of these marbles in a row? Three marbles? Four marbles?

5. Little Bear fell down from the honey tree. His friend Porcupine, an aspiring movie director, filmed his fall. Now Porcupine plans to add sound effects to his movie. Whenever Little Bear hits a tree branch, Porcupine plans to add an exclamation: either "Oooh!" or "Aaah!" or "Uuuh!"
 - a. How many different soundtracks can Porcupine create if during his fall Little Bear collides with 2 branches?
 - b. How about 3 branches? 4 branches? 5 branches?



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6. The alphabet of the planet Xork has four letters only: A, B, C, and D. The planet is divided into two kingdoms: Talkers and Chatterers.
 - a. A. In the kingdom of Chatterers, a first name can be any three-letter word (that is, letter combination) composed of the letters A, B, and C. How many different first names do they have in this kingdom?
 - b. In the same kingdom, a last name can be any four-letter word composed of letters A, B, C and D, so long as it ends in D. How many different last names do they have in this kingdom?
 - c. In the kingdom of Talkers, a first name can be any word of up to three letters composed of letters A, B, and C. How many different first names do they have in this kingdom?
 - d. In the same kingdom, a last name can be any four-letter word composed of letters A, B, C and D, so long as it has only one D in it, and this D is either the first or the last letter. How many different last names do they have in this kingdom?

7. Prince Ivan is on a quest to free Princess Masha, who has been imprisoned in the castle. The castle door has a simple digital lock with ten buttons, numbered 0 to 9. The door is guarded by a hungry dragon, Pashka, who likes hot dogs. The door lock can be opened by typing a secret 4-digit code, and Pashka can be distracted by hot dogs. It takes 1 second for Prince Ivan to try out a single 4-digit combination, and it takes 20 seconds for Pashka to gulp down a single hot dog. After Ivan opens the lock, it will take him one minute to fetch Masha and fly off on his magic carpet.
 - a. A. How many hot dogs should Ivan pack for the quest if he wants to fly out of the castle alive and with Masha? (Ivan should have enough hot dogs to keep Pashka at bay even in the worst case: when the secret code will be the last possible combination that he tries.)
 - b. Suppose that Ivan knows in advance that the secret 4-digit code is composed of odd digits only. How many hot dogs will he need now?
 - c. Suppose that Ivan knows in advance that the secret 4-digit code is composed of odd digits only and has exactly one digit 5 in it. How many hot dogs will he need in this case?



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8. There are only 6 letters in the Martian alphabet. All Martian words are exactly 4 letters long.
 - a. Suppose that any sequence of 4 letters is a valid Martian word. How many words are there in the Martian language?
 - b. Suppose that any sequence of 4 *non-repeating* letters is a valid Martian word. How many words are there in the Martian language?
 - c. Suppose that any sequence of 4 letters *that has at least one repetition* is a valid Martian word. How many words are there in the Martian Language? (There's an easy way to do this!)

9. Tim, Alex, Anton, and Eugenia are in charge of organizing a lottery for their school fundraiser. They plan to print 1000 lottery tickets and sell them at \$1 per ticket. The tickets are numbered 1 to 1000. The winning tickets will be announced at the fundraiser. Right now the team is discussing the criteria for the winning tickets.
 - Tim suggests that all tickets with numbers composed only of odd digits should win \$7 per ticket (for example, 11, 3, 199, 375, 111).
 - Alex proposes a different scheme: to award \$8 for any ticket that ends with the digit 1 (for example 3, 31, 191, 371, 111).
 - Anton favors the idea that every ticket that contains exactly one digit 3 will get \$3 (for example 3, 31, 345, 131, 293).

The boys start to argue: each one thinks that his plan is the best (will collect the most money if all the tickets are sold). Eugenia, who attends a math circle, does some calculations. Then she explains how much money each plan would bring. According to Eugenia, which plan is the most profitable?