TULSA GIRLS' MATH CIRCLE

Number Theory Questions II

Primes (after using the Sieve of Eratosthenes)

1. Find the smallest composite number not divisible by 2.
   Find the smallest composite number divisible by neither 2 nor 3.
   Find the smallest composite number not divisible by any of 2, 3, or 5.
   For a prime number *p*, find the smallest composite number that has no prime divisor less than *p*.

2. Each week, between 30 and 50 students show up for an art class run by Tina and Amy. Usually the students break up into groups of equal size. However, this week, Tina noticed that she could not break the students up into multiple groups of equal size. Amy noticed that if Tina and Amy joined the students, they still could not break the group into groups of equal size. How many students showed up to the art class this week?

3. What is the largest two-digit prime number whose digits are also each prime?

4. When determining whether or not a natural number is prime, is it necessary to test for divisibility by every integer between 1 and the natural number itself?

5. A group of 25 pennies is arranged into three piles such that each pile contains a different prime number of pennies. What is the greatest number of pennies possible in any of the three piles?

6. What is the smallest prime divisor of $5^{23} + 7^{17}$?

7. What are the 5 smallest prime numbers greater than 1000?

8. How many primes are there? How do you know?

Prime Factorization (after the Fundamental Theorem of Arithmetic)

9. Two numbers have respective prime factor decompositions $2 \times 3^2 \times 7^3 \times 13$ and $2 \times 3^2 \times 7^2$.
    a. Is the first number divisible by the second?
    b. Is the product of these numbers divisible by 8? By 36 by 27? By 16? By 56?

10. Using the prime factorization of the numbers, find:
    Gcd(80,144), gcd(160,288), gcd(240,432), gcd(80n,144n)
    Then find the Lcm(80,144), lcm(160,288), lcm(240,432), LCM (80n,144n)

11. If gcd(n,70) = 10 and lcm(n,70)=210. Find n

12. Janet writes the cubes of three positive integers on a piece of paper. Rob points out that each is a multiple of 18. Janet then points out that the GCD of all three perfect cubes is *n*. Find the smallest possible value of *n*.

Divisor Counting problems:

1. Find the prime factorization of 168.
    Find the number of positive divisors of 168.
    What do we know about the prime factorization of an even divisor of 168?
    How many of the positive divisors of 168 are even?

2. A certain integer has 20 positive divisors.
    What is the smallest number of primes that could divide the integer?
    What is the largest number of primes that could divide the integer?
    What is the smallest natural number that has exactly 20 positive divisors?

3. A little gnome was born in the year 1122. On every birthday, he receives a precious stone from his grandma: either a diamond or a ruby. His grandma presents him with a diamond on those years when the gnome's age is a factor of that year's number. She gives him rubies on all other birthdays. For example, the gnome received a diamond on his first birthday (since 1123 is divisible by 1), on his second birthday (since 1124 is divisible by 2), on his third birthday (since 1125 is divisible by 3) and so on. However, he received rubies on his 5^{th} birthday (since 1127 is not divisible by 5), on his 7^{th} birthday (since 1129 is not divisible by 7) and so on. How old was the gnome when he received his last birthday diamond? (Assume that both little gnomes and their grandmas live thousands and thousands of years.)

4. Solve: $2222 = A \times BB \times BCB$. The same letter always stands for the same digit, and different letters stand for different digits.

5. Let *p* be a prime number greater than 2. Is the number *p-1* odd or even? How about *p+1?*

6. Let *p* be a prime number greater than 2.
   a. PROVE that at least one of the numbers *p-1* and *p+1* is divisible by 4.
   b. What about divisibility by 5?