



One Pile, Two Plies, Three Piles *

The Tulsa Girls' Math Circle

ONE PILE

Rules: One pile is a two-player game. Place a small handful of stones in the middle. At every turn, a player decides whether to take one, two or three stones from the pile. Whoever takes the last stone(s) wins.

Try this game out with a partner a few times. You decide how many “stones” to put in the pile. Consider these questions, which we’ll explore more fully below, after you’ve played several games:

At what point of the game do you know if you’ve won or lost? Why? How many stones were in the pile when you knew? Does the first player always win? The second? Do you need to create a way to record your plays?

1. Suppose there are 4 stones in the pile. Do you want to go first or second? Explain why.
2. Suppose there are 9 stones in the pile. Do you want to go first or second? Explain why.
3. Suppose there are 15 stones in the pile. Do you want to go first or second? Explain why.
4. Can you come up with a full strategy for the game no matter how many stones are in the pile to start with? Write down your strategy in your notebook. Try it out on someone.
5. What if you can take up to 5 stones at each turn? What is the strategy now?

TWO PILES

Rules: We place two piles of stones in the middle. Players take turns. At each turn, you get to pick a pile and take as many stones as you want from that pile. Whoever takes the very last stone(s) wins. (Example: I could take one whole pile, but then my opponent could take the other pile and I would lose.)

Try this game out with a partner a few times. You decide how many “stones” to put in each pile. Consider these questions, which we’ll explore more fully below, after you’ve played several games:

At what point of the game do you know if you’ve won or lost? Why? How many stones were in each pile when you knew? Does the first player always win? The second? Can you create a situation where you know you will lose if you are the next player? Do you need to create a way to record your plays?

*See “Discovering The Art of Mathematics”, <https://www.artofmathematics.org/books/games-and-puzzles>



TWO PILES (continued)

1. Suppose there are 2 stones in each pile. Do you want to go first or second? Explain why.
2. Suppose there are 7 and 9 stones in the respective piles. Do you want to go first or second? Explain why.
3. Suppose there are 6 and 25 stones in the respective piles. Do you want to go first or second? Explain why.
4. Explain the full strategy of the game. Again, record it in your notebook. Diagrams, or illustrations are a wonderful part of any explanation.

Let's explore different ways to record plays and analyze the game with a game tree, a table.

THREE PILES

Rules: We place three piles of stones in the middle. Two players take turns. At each turn, you get to pick a pile and take as many stones as you want from that pile. Whoever takes the very last stone(s) wins.

Try this game out with a partner a few times. You decide how many "stones" to put in each pile. Consider these questions, which we'll explore more fully below, after you've played several games:

At what point of the game do you know if you've won or lost? Why? How many stones were in each pile when you knew? Does the first player always win? The second? Can you create a situation where you know you will lose if you are the next player? Do you need to create a way to record your plays?

1. Analyze these situations and explain whether you want to go first or second and why. Keep track of the moves and see if you can form a strategy.
 - a. Piles of 1, 2, and 2
 - b. Piles of 1, 2, and 3
 - c. Piles of 1, 2, and 4
2. We'll do a class example with 3 piles of 11, 9, and 6.
3. If you feel like you have a strategy, see if you can write it down in your notebook. Feel free to use diagrams or any other illustrations to help explain your strategy.
4. Try your strategy out with your partner with 3 piles of 10, 7, and 5 stones and with 8, 7, and 4 stones
5. What about more piles?!
6. What other questions can you come up with?