1. Parity game with 7 cups. Three upside-down (bottom up), four right-side up (bottom down). The order doesn’t matter. We will turn over two cups at a time by these rules:
   a. Two bottom-up cups should both be overturned and placed back on the table bottom-down.
   b. Two bottom-down cups should both be turned bottom-up.
   c. A pair with a bottom-up and a bottom-down cup should be placed back bottom-down and bottom-up respectively.

The object of the game is to get all seven cups bottom-down.

2. You have 5 two-color disks. They are all yellow on one side and red on the other. Your friend lays out the 5 disks with the red side up on all five. She then hides the disks and makes 4 flips (flip = turning any disk over once; the same disk can get flipped more than once, as long as there are 4 flips total). Your friend shows you 4 of the disks. Can you predict if the hidden disk is red side up or yellow side up?

3. Penny and Dime Magic Trick:
   (1) Place one coin in each hand.
   (2) Multiply the value of the coin in your left hand by any odd number.
   (3) Multiply the value of the coin in your left hand by any even number.
   (4) Add together the results from steps (2) and (3).
   (5) Predict which hand your friend’s penny is in based on their sum.

For all these games, can you explain WHY your conclusion has to be true?

References: Problem 1, *Mathematical Circle Diaries, Year 1*, by Anna Burago
             Problems 2, 3 [https://www.mathcircles.org/files/ParityProblems_0.pdf]