



TULSA GIRLS'  
MATH CIRCLE

## Tulsa Girls' Math Circle

### Parity II

Supplies: Chessboards Checkers Small craft sticks
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1. Can a  $5 \times 5$  chessboard be tiled by  $2 \times 1$  dominoes? (To tile means to cover the board completely, without any overlaps.)
2. On Tuesday morning, there were 5 space jets at the space port of the planet Pandora. During that day, several more jets landed, and a few jets departed from the port. The jets always arrived in groups of 2 or 4, and departed in groups of 2. In the evening, the air dispatcher counted 60 jets at the space port. Prove that the dispatcher miscalculated the number of jets.
3. Stephanie and her friends stand in a circle. It turns out that both neighbors of each person are of the same gender. If there are five boys in the circle, how many girls are there?
4. The integers from 1 to 18 are written on the board in a row. Can you insert plus and minus signs between them in such a way as to get an expression that is equal to 0? If so, show how. Can you insert plus and minus signs to get 0 if you the integers range from 1 to 17? If so, show how.
5. Five checkers are placed on a  $5 \times 5$  checkerboard in such a way that their positions are symmetric with respect to one of its diagonals. Prove that at least one of the checkers is positioned on that diagonal.
6. Can we draw a closed path made up of 9 line segments, each of which intersects exactly one of the other segments?

7. Three types of magic fruit—apples of wisdom, pears of bravery, and plums of kindness—grow on the Magic Tree in the center of the Far Away Kingdom. From time to time, some of these fruits are harvested for the benefit of the Kingdom. The Magic Tree immediately regrows the picked fruit according to the following set of rules:

- If a single fruit is picked from the tree, another of the same kind grows in its place.
- If 2 apples are picked, 4 pears grow back.
- If 2 pears are picked, 4 plums grow back.
- If 2 plums are picked, 4 apples grow back.
- If 2 fruits of different kinds are picked, nothing else happens.

Currently the tree has 11 apples, 10 pears and 8 plums. The wicked witch plans to weaken the Kingdom by stealing all the fruit. She intends to sneak to the tree several mornings in a row and pick one or two fruits every time. Is there a way for her to pick all the fruits of the tree? Either show how or explain why not.

8. Annie placed a full set of domino tiles (28 pieces) into a line according to the rules of dominoes. The left square of the leftmost tile has 6 spots on it. How many spots are there in the right square of the rightmost tile?

(Reminder: dominoes are rectangular tiles divided into two halves (squares). Each square has several spots marked on it; the number of spots ranges from 0 to 6. A 28-piece domino set contains tiles with all possible combinations of spots: [0-0], [0-1], [0-2], . . . , [0,6], [1-1], [1,2], . . . , [6-6]. In regular dominoes play, the adjacent ends of each pair of tiles should match (have an identical number of spots).

Problems 3, 5, 6 from *Math Circles (Russian Experience)* by Dmitri Fomin, et al