

TILINGS at Berkeley Math Circle!

Inspired by Activities of Julia Robinson Math Festival and Nina Cerutti and Leo B. of SFMC.



Directed By Joshua Zucker

Tiling Torment

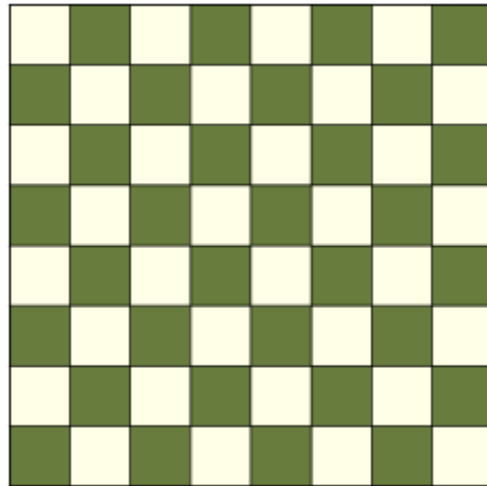
The problem

There are many problems that involve tiling (covering) all the squares on a chessboard (or similar board) with tiles of various sizes. The chessboard may be 8×8 , 7×7 or other sizes and may or may not have squares missing. The tiles can be dominoes (2×1) or tiles of other sizes.

Questions

The Basics

1. Is it possible to tile a 7×7 board with 2×1 tiles?
2. In general, is it possible to tile an $n \times n$ board with 2×1 tiles? If so, which boards can you tile and why?



Taking it Further

3. Now consider the 7×7 board again. If you remove one square, is it possible to tile the board? If so, does it matter which square you remove? Describe completely.
4. In general, if n is odd, is it possible to tile an $n \times n$ board with 2×1 tiles if one square is covered with a 1×1 tile? Does it matter which square is covered?
5. Remove two diagonally opposite corners of a chessboard. Is it possible to tile this shape with 31 2×1 tiles?
6. In general, if n is even, is it possible to tile an $n \times n$ board with 2×1 tiles if two squares are removed? Does it matter which two squares are removed?
7. Is it possible to tile an 8×8 board with 21 “L-shaped” tiles of three squares and one 1×1 tile? If so, how? Describe all possible locations for the 1×1 tile. If not, why not?