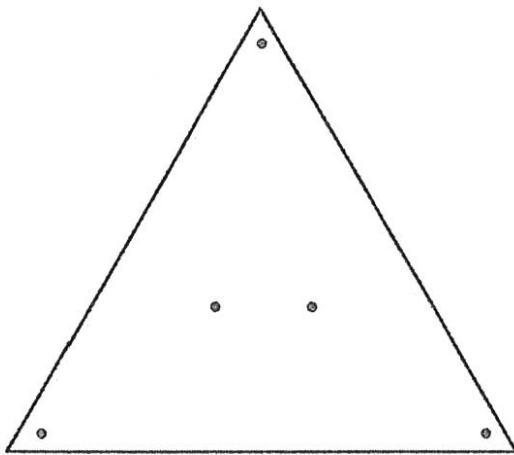


The Game of Criss-Cross
(Euler Characteristic)

From: *Circle in a Box* by Sam Vandervelde (MSRI Mathematical Circles Library)

The game of Criss-Cross is played on a blank sheet of paper by two players. The game board is created by drawing three points at the vertices of a large equilateral triangle, along with two to seven additional points anywhere in its interior. Players alternate turns drawing a single straight line segment joining any two points, as long as the segment does not pass through any other points or segments already appearing on the game board. The winner is the last player able to make a legal move.



How many different moves can the first player make on this game board?

Will the first or second player win?

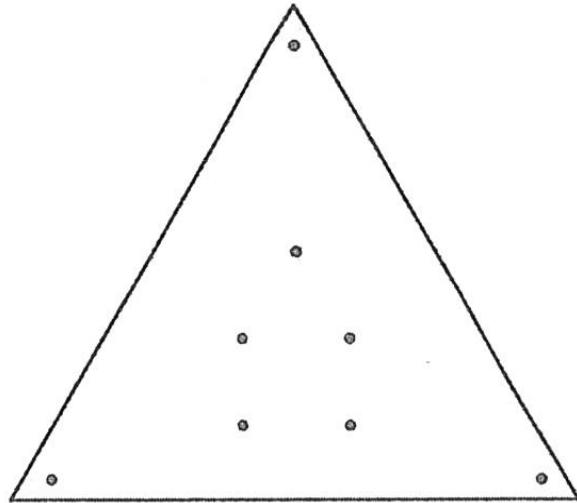
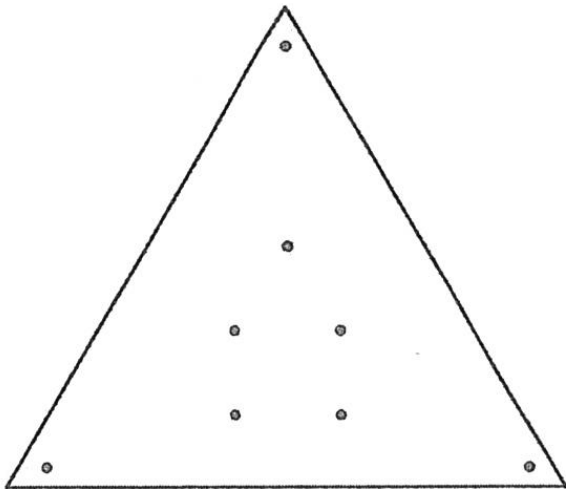
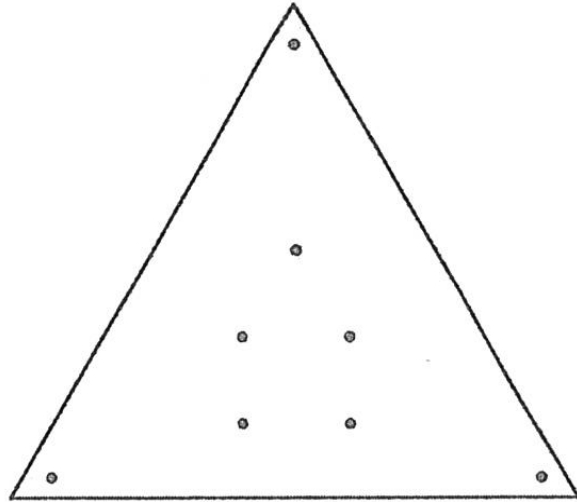
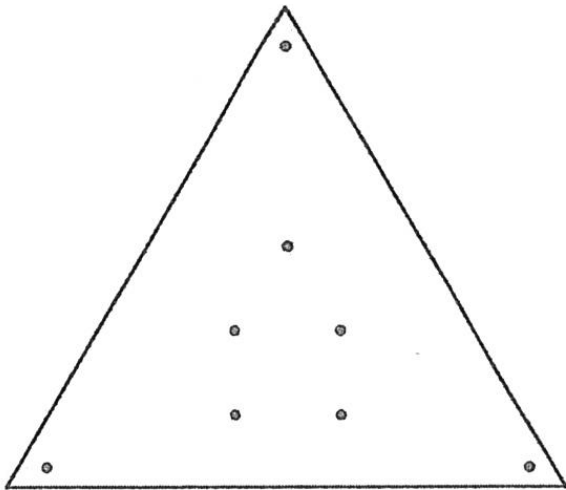
Play several games on the board with 8 initial vertices and come up with a method for predicting the winner of any game of Criss-Cross based on the board configuration.

For each of the games you play, count the number of vertices (points), edges (segments), and faces (regions) appearing in the completed game board. Be sure to include the area surrounding the game as one of your regions.

Vertices (V)	Edges (E)	Faces (F)

Compare the number of edges and faces on each completed game board, then make a conjecture about these two quantities. Prove your conjecture. (First, why are all the regions triangular?)

The expression $V-E+F$ is the Euler characteristic. Prove that the Euler characteristic of any completed game board is 2.



Use the relationship between V , E , and F to predict the number of edges and faces that will appear on a completed game board which starts with a total of 99 points. Which player wins? Prove your method of predicting the winner on any game board is valid.